

Mark Scheme Gravity Fields Past Paper Questions Jan 2002—Jan 2010 (old spec)

2

- (a) period = 24 hours or equals period of Earth's rotation ✓
 remains in fixed position relative to surface of Earth ✓
 equatorial orbit ✓
 same angular speed as Earth or equatorial surface ✓

Q2 Jun 2003

max(2)

(b)(i) $\frac{GMm}{r^2} = m\omega^2 r$ ✓

$$T = \frac{2\pi}{\omega} \quad \checkmark$$

$$r \left(= \frac{GMT^2}{4\pi^2} \right)^{1/3} = \left(\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times (24 \times 3600)^2}{4\pi^2} \right)^{1/3} \quad \checkmark$$

(gives $r = 42.3 \times 10^3$ km)

(b)(ii) $\Delta V = GM \left(\frac{1}{R} - \frac{1}{r} \right)$ ✓

$$= 6.67 \times 10^{-11} \times 6 \times 10^{24} \times \left(\frac{1}{6.4 \times 10^6} - \frac{1}{4.23 \times 10^7} \right) = 5.31 \times 10^7 \text{ (J kg}^{-1}\text{)} \quad \checkmark$$

$$\Delta E_p = m\Delta V (= 750 \times 5.31 \times 10^7) = 3.98 \times 10^{10} \text{ J} \quad \checkmark$$

(allow C.E. for value of ΔV)

[alternatives:

calculation of $\frac{GM}{R}$ (6.25×10^7) or $\frac{GM}{r}$ (9.46×10^6) ✓

or calculation of $\frac{GMm}{R}$ (4.69×10^{10}) or $\frac{GMm}{r}$ (7.10×10^9)

calculation of both potential energy values ✓

subtraction of values or use of $m\Delta V$ with correct answer ✓]

(6)

(8)

- (a) work = force \times distance moved in direction of force ✓
 (in circular motion) force is perpendicular to displacement ✓
 no movement in direction of force ✓ (hence no work)
 [or speed of body remains constant (although velocity changes) ✓
 kinetic energy is constant ✓
 potential energy is constant ✓]

[or gravitational force acts towards the Earth ✓
 Moon remains at constant distance/radius from Earth ✓
 since radius is unchanged, gravitational force does no work
 or E_p of Moon is constant ✓]

(3)

- (b)(i) any suitable example of circular motion ✓

- (ii) any SHM example at maximum displacement ✓
 [or any other suitable example, e.g. car starts from rest]

(2)

(5)

Quality of Written Communication: Q3(a) and Q4(a) ✓✓

(2)

(2)

Question 3

- (a) work done/energy change (against the field) per unit mass ✓
 when moved from infinity to the point ✓

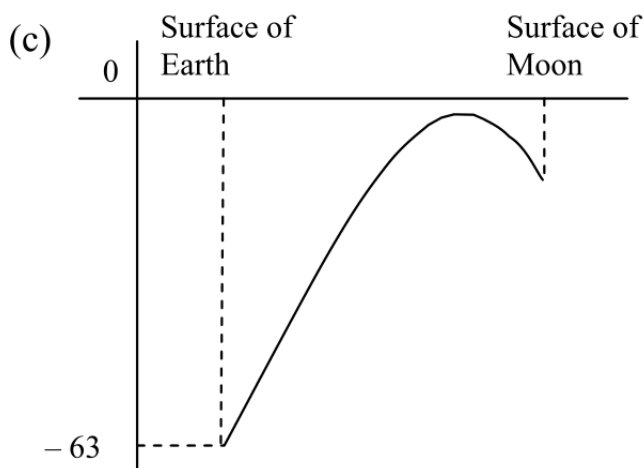
(2)

Q3 Jan 2005

$$(b) \quad V_E = -\frac{GM_E}{R_E} \quad \text{and} \quad V_M = -\frac{GM_M}{R_M} \quad \checkmark$$

$$V_M = -G \times \frac{M_E}{81} \times \frac{3.7}{R_E} = \frac{3.7}{81} V_E \quad \checkmark$$

$$= 4.57 \times 10^{-2} \times (-63) = -2.9 \text{ MJ kg}^{-1} \quad \checkmark \quad (2.88 \text{ MJ kg}^{-1}) \quad (3)$$



limiting values $(-63, -V_M)$

on correctly curving line ✓

rises to value close to but below zero ✓

falls to Moon ✓

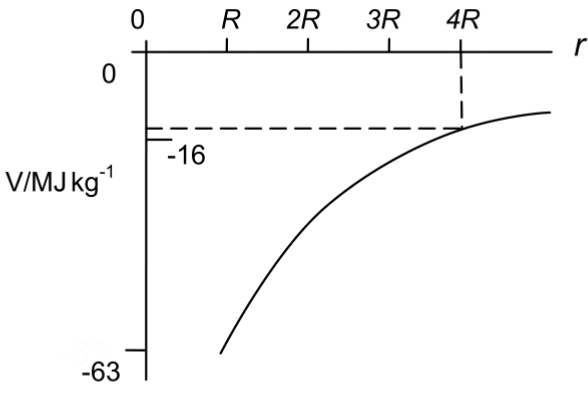
from point much closer to M than E ✓

max(3)

(8)

Question 4	Q4 Jun 2005	
(a)	attractive force between point masses ✓ proportional to (product of) the masses ✓ inversely proportional to square of separation/distance apart ✓	3
(b)	$m\omega^2 R = (-)\frac{GMm}{R^2} \left(\text{or} = \frac{mv^2}{R} \right) \checkmark$ (use of $T = \frac{2\pi}{\omega}$ gives) $\frac{4\pi^2}{T^2} = \frac{GM}{R^3} \checkmark$ G and M are constants, hence $T^2 \propto R^3 \checkmark$	3
(c) (i)	(use of $T^2 \propto R^3$ gives) $\frac{365^2}{(1.50 \times 10^{11})^3} = \frac{T_m^2}{(5.79 \times 10^{10})^3} \checkmark$ $T_m = 87(.5)$ days ✓ (ii) $\frac{1^2}{(1.50 \times 10^{11})^3} = \frac{165^2}{R_N^3} \checkmark$ (gives $R_N = 4.52 \times 10^{12}$ m) ratio = $\frac{4.51 \times 10^{12}}{1.50 \times 10^{11}} = 30(.1) \checkmark$	4

Question 4	Q4 Jan 2006	
(a)	orbits (westwards) over Equator ✓ maintains a fixed position relative to surface of Earth ✓ period is 24 hrs (1 day) or same as for Earth's rotation ✓ offers uninterrupted communication between transmitter and receiver ✓ steerable dish not necessary ✓	Max 3
(b) (i)	$G \frac{Mm}{(R+h)^2} = m\omega^2(R+h) \checkmark$ (ii) use of $\omega = \frac{2\pi}{T} \checkmark$ gives $\frac{GM}{(R+h)^3} = \frac{4\pi^2}{T^2}$, hence result ✓ (iii) limiting case is orbit at zero height i.e. $h = 0 \checkmark$ $T^2 = \left(\frac{4\pi^2 R^3}{GM} \right) = \frac{4\pi^2 \times (6.4 \times 10^6)^3}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}} \checkmark$ $T = 5090$ s ✓ (= 85 min)	6
(c)	speed increases ✓ loses potential energy but gains kinetic energy ✓ [or because $v^2 \propto \frac{1}{r}$ from $\frac{GMm}{r^2} = \frac{mv^2}{r}$] [or because satellite must travel faster to stop it falling inwards when gravitational force increases]	2
	Total	11

Question 4		
(a)	force per unit mass ✓ [or force on a 1 kg mass or $g = F/m$ with terms explained] vector ✓	2
(b)	<p>(i) $F = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 6.00 \times 10^{24} \times 2.5 \times 10^3}{(1.6 \times 10^7)^2}$ ✓ Q4 Jan 2007</p> <p>= 3900 N (3910) ✓</p> <p>(ii) $V_{\text{orbit}} \left(= -\frac{GM}{r} \right) = -\frac{6.67 \times 10^{-11} \times 6.00 \times 10^{24}}{1.6 \times 10^7}$ ✓</p> <p>= -25 (MJ kg⁻¹) (-25.0) ✓</p> <p>[or $\frac{V_{\text{orbit}}}{V_{\text{surface}}} = \frac{r_{\text{surface}}}{r_{\text{orbit}}}$ ✓</p> <p>gives $V_{\text{orbit}} = -\left(\frac{6.4 \times 10^6}{1.6 \times 10^7} \right) \times 63 = -25 \text{ (MJ kg}^{-1}\text{)} (-25.2)$ ✓]</p> <p>$\Delta V = (63 - 25) \times 10^6 = 38 \times 10^6 \text{ (J kg}^{-1}\text{)}$ ✓</p> <p>$\Delta E_p (= m \Delta V) = 2.5 \times 10^3 \times 38 \times 10^6 = 9.5 \times 10^{10} \text{ J}$ ✓</p>	max 5
(c)	<p>line starts at $(R, -62.5)$ and ends at a finite value ✓</p> <p>curve of decreasing positive gradient ✓</p> <p>correct $(1/r)$ relationship shown by axis values ✓</p> 	3
	Total	10

Question 5		
(a)	<p style="text-align: right;">Q5 Jan 2008</p> $\frac{GMm}{r^2} = m\omega^2 r \text{ (or } = \frac{mv^2}{r} \text{)} \checkmark$ <p>correct application of $T = \frac{2\pi}{\omega}$ (or $v = \frac{2\pi r}{T}$) \checkmark</p> <p>(gives $\frac{GMm}{r^2} = \frac{4\pi^2 mr}{T^2}$ and $T^2 = \frac{4\pi^2 r^3}{GM}$)</p> <p>in which m has cancelled or does not appear in expression \checkmark</p>	3
(b)	<p>(i) $\omega \left(= \frac{2\pi}{T} \right) = \frac{2\pi}{7.15 \times 24 \times 3600} = 1.0(2) \times 10^{-5} \text{ (rad) s}^{-1} \checkmark$</p> <p>(ii) $\omega^2 r (1.02 \times 10^{-5})^2 \times 1.07 \times 10^9 = 0.11(1) \text{ m s}^{-2} \checkmark$</p> <p>(iii) centripetal acceleration = g (or $\alpha = \frac{GM}{r^2}$) \checkmark</p> $M \left(= \frac{gr^2}{G} \right) = \frac{0.111 \times (1.07 \times 10^9)^2}{6.67 \times 10^{-11}} \checkmark = 1.9(1) \times 10^{27} \text{ kg } \checkmark$ <p>[or use of $T^2 = \frac{4\pi^2 r^3}{GM}$ with $T = 6.18 \times 10^5 \text{ s } \checkmark$</p> $\text{gives } M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 \times (1.07 \times 10^9)^3}{6.67 \times 10^{-11} \times (6.18 \times 10^5)^2} \checkmark$ <p style="text-align: right;">$= 1.9(0) \times 10^{27} \text{ kg } \checkmark]$</p>	max 5
	Total	8

Question 3		
(a)	(i)	gravitational force (or field) decreases as r increases ✓ gravitational force (or field strength) is proportional to $(1/r^2)$ ✓ [award both marks for second statement alone] Q3 Jun 2008
	(ii)	mass of Moon $M \left(= \frac{Fr^2}{Gm} \right) = \frac{1600 \times (1.75 \times 10^6)^2}{6.67 \times 10^{-11} \times 1000} \checkmark$ $= 7.3(5) \times 10^{22} \text{ kg } \checkmark$ [or by use of any other consistent values of F and r]
(b)	(i)	E_p lost = area under graph ✓ acceptable method for finding area and values ✓ acceptable value for E_p lost ✓ [allow $(2.8 \pm 1.0) \times 10^9 \text{ J}$] [alternative mark scheme, for candidates who use values from the graph: potential of Moon's surface $= \left(-\frac{GM}{r} \right) = -\frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{1.75 \times 10^6} = -2.80 \times 10^6 \text{ (J kg}^{-1}\text{)} \checkmark$ change in potential $\Delta V = (-2.80 \times 10^6) - 0$ $= (-)2.80 \times 10^6 \text{ (J kg}^{-1}\text{)} \checkmark$ potential energy lost ($= m \Delta V$) = $1000 \times 2.80 \times 10^6$ $= 2.80 \times 10^9 \text{ J } \checkmark]$
	(ii)	$\frac{1}{2}mv^2 = 2.8 \times 10^9$ (or the E_p value from (b) (i)) ✓ gives escape speed $v = 2370 \text{ m s}^{-1}$ (or a consistent value) ✓ [alternative mark scheme, for candidates who use gravitational potential equation: $\frac{1}{2}mv^2 = \frac{GMm}{r}$ gives $v = \sqrt{\frac{2GM}{r}}$ $= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{1.75 \times 10^6}} \checkmark$ $= 2370 \text{ m s}^{-1} \checkmark]$
		Total
		9

Question 3		
(a)	<p style="text-align: right;">Q3 Jun 2009</p> <p>(i) gradient $\left(= \frac{5.9 \times 10^8}{6000} \right) = 9.83 \times 10^4 \text{ (ms}^{-2/3}) \checkmark$</p> <p>(for 9.83 allow 9.7 to 10.0)</p> <p>(ii) cube root of equation is $R = \left(\frac{GM}{4\pi^2} \right)^{1/3} T^{2/3}$</p> <p>(or equation predicts $R \propto T^{2/3}$) \checkmark</p> <p>$R \propto T^{2/3}$ confirmed by graph as a straight line through (0, 0) (or a line of constant gradient through (0, 0)) \checkmark</p> <p>(iii) use of gradient of graph as $\left(\frac{GM}{4\pi^2} \right)^{1/3}$ or $\left(\frac{R}{T^{2/3}} \right) \checkmark$</p> <p>$\left(\frac{GM}{4\pi^2} \right)^{1/3} = 9.83 \times 10^4$ gives $\left(\frac{GM}{4\pi^2} \right) = 9.50 \times 10^{14} \text{ (m}^3 \text{s}^{-2}) \checkmark$</p> <p>mass of Saturn $M = \frac{9.50 \times 10^{14} \times 4\pi^2}{6.67 \times 10^{-11}} = 5.62 \times 10^{26} \text{ kg} \checkmark$</p>	6
(b)	<p>similarity:</p> <p>graph would also be a straight line (through (0, 0) because $R \propto T^{2/3}$ (or $R^3 \propto T^2$) always applies to any satellite \checkmark</p> <p>difference:</p> <p>gradient would be <i>larger</i> because mass of Sun > mass of Saturn \checkmark</p>	2
	Total	8