Mark Scheme SHM Paper Questions
Jan 2002—Jan 2010 (old spec)

Q1 Jun 2002

1(a) forced vibrations or resonance ✓ (*1*)

(b) reference to natural frequency (or frequencies) of structure ✓
driving force is at same frequency as natural frequency of structure ✓
resonance ✓
large <u>amplitude</u> vibrations produced or large energy transfer to structure✓
could cause damage to structure [or bridge to fail] ✓ $_{\text{max}}$(*4*)

(c) stiffen the structure (by reinforcement) ✓
install dampers or shock absorbers ✓
[or other acceptable measure e.g. redesign to change natural frequency
or increase mass of bridge or restrict number of pedestrians] (*2*)

(*7*)

Q2 Jan 2002

2(a) use of $mg = ke$ gives $k = \dfrac{0.20 \times 9.81}{3.5 \times 10^{-2}}$ ✓

$= 56\ \text{N m}^{-1}$ ✓ [or kg s^{-2}] (*2*)

(b)(i) $28\ (\text{N m}^{-1})$ ✓ (unit to be given in either (a) or (b))
(allow C.E. from (a))

(ii) (use of $T = 2\pi\sqrt{\dfrac{m}{k}}$ gives) $T = 2\pi\sqrt{\dfrac{0.50}{28}} = 0.84\ (\text{s})$ ✓
(allow C.E. for value of k from (b)(i))

number of oscillations per minute $= \dfrac{60}{0.84} = 71$ ✓
(allow C.E. from (b)(ii)) (*3*)

(*5*)

1(a) (use of $T = 2\pi\sqrt{\frac{l}{g}}$ gives) $T = 2\pi\sqrt{\frac{0.80}{9.81}}$ ✓ **Q1 Jan 2003**

$= 1.8$ s ✓ (*2*)

(b) $mgh = ½\, mv^2$ ✓

$v = \sqrt{(2 \times 9.81 \times 20 \times 10^{-3})}$ ✓ (= 0.63 m s^{-1})

$v_{max} = 2\pi fA = \frac{2\pi A}{T}$ ✓

$A = \frac{0.63 \times 1.8}{2\pi}$ ✓ (= 0.18 m)

[or by Pythagoras $A^2 + 780^2 = 800^2$ ✓

gives A = $\sqrt{(800^2 - 780^2)}$ ✓ (= 180 mm)

(or equivalent solution by trigonometry ✓ ✓)

$v_{max} = 2\pi fA$ or $= \frac{2\pi A}{T}$ ✓

$= \frac{2\pi \times 0.18}{1.8}$ ✓ (= 0.63 m s^{-1})] (*4*)

(c) tension given by F, where $F - mg = \frac{mv^2}{l}$ ✓

$F = 25 \times 10^{-3}\left(9.81 + \frac{0.63^2}{0.8}\right) = 0.26$ N ✓ (*2*)

(*8*)

Question 1 **Q1 Jan 2005**

(a) acceleration is proportional to displacement ✓
acceleration is in opposite direction to displacement, or
towards a fixed point, or towards the centre of oscillation ✓ (*2*)

(b)(i) $f = \frac{25}{23} = 1.1$ Hz (or s^{-1}) ✓ (1.09 Hz)

(ii) (use of $a = (2\pi f)^2 A$ gives) $a = (2\pi \times 1.09)^2 \times 76 \times 10^{-3}$ ✓
$= 3.6$ m s^{-2} ✓ (3.56 m s^{-2})
(use of f = 1.1 Hz gives a = 3.63 m s^{-2})
(allow C.E. for incorrect value of f from (i))

(iii) (use of $x = A\cos(2\pi ft)$ gives) $x = 76 \times 10^{-3} \cos(2\pi \times 1.09 \times 0.60)$ ✓
$= (-)4.3(1) \times 10^{-2}$ m ✓ (43 mm)
(use of f = 1.1 Hz gives $x = (-)4.0(7) \times 10^{-2}$ m (41 mm))
direction: above equilibrium position or upwards ✓ (*6*)

(c)(i) graph to show:
correct shape, i.e. cos curve ✓
correct phase i.e. –(cos) ✓

(ii) graph to show:
two cycles per oscillation ✓
correct shape (even if phase is wrong) ✓
correct starting point (i.e. full amplitude) ✓ max(*4*)

(*12*)

Q1 Jan 2006

Question 1			
(a)	(i)	$mg = ke$ ✓ $k = \left(\frac{0.25 \times 9.81}{40 \times 10^{-3}}\right) = 61(.3)$ N m^{-1} ✓	
	(ii)	$T\left(= 2\pi\sqrt{\frac{m}{k}}\right) = 2\pi\sqrt{\frac{0.69}{61.3}}$ ✓ (= 0.667 s) $f\left(= \frac{1}{T}\right) = \frac{1}{0.667}$ ✓ (= 1.50 Hz)	**4**
(b)	(i)	forced vibrations (at 0.2 Hz) ✓ amplitude less than resonance (≈ 30 mm) ✓ (almost) in phase with driver ✓	
	(ii)	resonance [or oscillates at 1.5 Hz] ✓ amplitude very large (> 30 mm) ✓ oscillations may appear violent ✓ phase difference is 90º ✓	**Max 6**
	(iii)	forced vibrations (at 10 Hz) ✓ small amplitude ✓ out of phase with driver [or phase lag of (almost) π on driver] ✓	
		Total	**10**

Q1 Jan 2009

Question 1		
(a)	shm is defined by acceleration ∝ displacement (from mean position) ✓ explanation of – sign ✓ (e.g. acceleration is in opposite direction to displacement, or is always directed towards a fixed point, or towards equilibrium position)	**2**
(b) (i)	$T\left(=2\pi\sqrt{\frac{l}{g}}\right)=2\pi\sqrt{\frac{0.64}{9.81}}$ ✓ gives T = 1.60 (s) ✓ time for bob to travel from **A** to **C** = $T \div 4$ = 0.40 s ✓	
(ii)	max speed of bob v_{max} = 2 π $f A$ ✓ $= \frac{2\pi \times 44 \times 10^{-3}}{1.60}$ ✓ (= 0.173 m s^{-1}) max E_K of bob (= ½ $m\, v_{max}^2$) = ½ × 1.5 × 10^{-2} × 0.173^2 ✓ = 2.2(4) × 10^{-4} J ✓ [**or** max E_K of bob = E_P gained in moving from **C** to **B** ✓ (1280 - Δh) Δh = 442 [or 6402 = 442 + $(640 - \Delta h)^2$] gives Δh = 1.52 (mm) ✓ max E_K of bob (= $mg\Delta h$) = 1.5 × 10^{-2} × 9.81 × 1.52 × 10^{-3} ✓ = 2.2(4) × 10^{-4} J ✓]	**7**
	Total	**9**

Q1 Jan 2010

Question 1		
(a)	$mg = ke$ ✓ $k = \frac{0.20 \times 9.81}{43 \times 10^{-3}}$ = 46 N m^{-1} ✓ (45.6)	**2**
(b) (i)	new spring constant = 23 N m^{-1} ✓ (22.8) [unit of N m^{-1} or kg s^{-2} to appear in either (a) or (b) (i)]	
(ii)	period $T\left(=2\pi\sqrt{\frac{m}{k}}\right) = 2\pi\sqrt{\frac{0.50}{22.8}}$ = 0.930 (s) ✓ number of oscillations per minute = $\frac{60}{0.930}$ = 65 ✓ (64.5) [In (b) (ii), answer should be consistent with use of 0.50 kg and candidate's answer to (b) (i)]	**5**
	Total	**5**

Question 1	**Q1 Jun 2005**	
(a)	reference to resonance ✓ air set into vibration at frequency of loudspeaker ✓ resonance when driving frequency = natural frequency of air column ✓ more than one mode of vibration ✓ stationary wave (in air column) ✓ (or reference to nodes and antinodes) maximum amplitude vibration (or max energy transfer) at resonance ✓ [alternative answer to (a): first two marks as above, remaining four marks for wave reflected from surface (of water) ✓ interference/superposition (between transmitted and reflected waves) ✓ maximum intensity when path difference is $n\lambda$ ✓ maxima (or minima) observed when l changes by $\lambda/2$ ✓]	**Max 4**
(b) (i)	$\frac{\lambda}{2} = 523 - 168$ ✓ (= 355 mm) $\lambda = 710$ mm ✓ [if $\frac{\lambda}{4} = 168$, giving $\lambda = 670$ mm, ✓ (1 max) (672 mm)]	**4**
(ii)	$c(= f\lambda) = 480 \times 0.71$ ✓ $= 341 \text{ m s}^{-1}$ ✓ (allow C.E. for incorrect λ from (i)) [allow $480 \times 0.67 = 320 \text{ m s}^{-1}$ ✓ (1max) (322 m s^{-1})]	

Question 1		
(a)	$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ ✓ **Q1 Jun 2006** oscillations must be of small amplitude ✓	**2**
(b) (i)	$f = \frac{25}{46.5}$ =0.53(8) (s^{-1}) ✓ [or $T = \frac{46.5}{25}$ = 1.8(6) (s)] $l\left(=\frac{g}{4\pi^2 f^2}\right) = \frac{9.81}{4\pi^2 0.538^2}$ [or $l\left(=\frac{T^2 g}{4\pi^2}\right) = \frac{1.86^2 \times 9.81}{4\pi^2}$] ✓ $l = 0.85(9)$ m ✓ (allow C.E. for values of f or T)	
(ii)	$a_{max}\{= (-)(2\pi f)^2 A\} = (2\pi \times 0.538)^2 \times 51 \times 10^{-3}$ ✓ (= 0.583 m s^{-2}) (allow C.E. for value of f from (i)) $F_{max}(= ma_{max}) = 1.2 \times 10^{-2} \times 0.583$ ✓ $= 7.0 \times 10^{-3}$ N ✓ (6.99×10^{-3} N) [or $F_{max}(= mg \sin \theta_{max})$ where $\sin \theta_{max} = \frac{51}{859}$ ✓ $= 1.2 \times 10^{-2} \times 9.81 \times \frac{51}{859}$ ✓ $= 6.99 \times 10^{-3}$ N ✓]	**6**
	Total	**8**

Question 2	Q2 Jun 2006	
(a)	vibrates or oscillates or moves in shm ✓ vibration/oscillation is vertical/perpendicular to wave propagation direction ✓ frequency ($=c/\lambda$) = 3.0 (Hz) ✓ (or same as P) amplitude = 90 (mm) ✓ (or same as P) Q has a phase lag on P ✓ (or vice versa) phase difference of $\left(\frac{0.4}{1.2}\times 2\pi\right)=\frac{2\pi}{3}$ (rad) or 120° ✓	**max 5**
(b)	use of f = 3.0 (Hz) ✓ v_{max} ($= 2\pi fA$) $= 2\pi \times 3.0 \times 90 \times 10^{-3}$ ✓ = 1.7(0) m s^{-1} ✓	**3**
	Total	**8**

Question 1		Q1 Jun 2007	
(a)	(i)	P at any peak or trough ✓ Q at any point where velocity is zero and slope is negative ✓	
	(ii)	R at any point where velocity is zero ✓ acceleration is gradient of v/t graph which is a maximum at R [**or** in SHM acceleration is greatest when velocity is zero (or equivalent statement)] ✓	**4**
(b)	(i)	$mg = ke \therefore$ static extension $e = \frac{0.40 \times 9.81}{28}$ = 0.14(0) m ✓	
	(ii)	total extension = 0.140 + 0.060 = 0.200 m ✓ energy stored (= ½ F e) = ½ × (28 × 0.200) × 0.200 ✓ = 0.56 J ✓ (allow ✓ for use of ½ $F e$ if incorrect value is taken for e) **or** E_P stored at equilibrium (= ½ Fe) = ½ × (28 × 0.14) × 0.14 = 0.274 (J) ✓ maximum E_K of oscillating mass (= ½ m $(2\pi f A)^2$) = 0.050 (J) ✓ total E_P stored = 0.274 + 0.050 + $m g A$ = 0.324 + (0.40 × 9.81 × 60 × 10^{-3}) = 0.324 + 0.235 = 0.56 J ✓	**4**
		Total	**8**

Question 1			
(a)		$T\left(=2\pi\sqrt{\frac{m}{k}}\right)$ gives $\frac{1}{0.92} = 2\pi\sqrt{\frac{400}{k}}$ ✓ **Q1 Jun 2008** from which k = 1.3(4) × 10^4 N m^{-1} ✓ [or by use of effective spring constant for all four springs: springs in parallel so k' = 4k for a total mass of 1600 kg]	**2**
(b)	(i)	when t = 0.20 s $x(= A\cos 2\pi ft)$ = 90 cos 2π (0.92 × 0.20) ✓ gives x = 36(.3) mm ✓ downwards ✓	**max 4**
	(ii)	vertical speed v (= $2\pi f\sqrt{A^2 - x^2}$) = 2π × 0.92 $\sqrt{((90\times10^{-3})^2 - (36.3\times10^{-3})^2)}$ ✓ [**or** v = ($-2\pi fA\sin 2\pi ft$) = (-) 2π × 0.92 × 90 × 10^{-3} sin (2π × 0.92 × 0.20) ✓] gives v = 0.47(6) m s^{-1} ✓	
(c)		same period maintained throughout graph ✓ exponential decay of amplitude ✓	**2**
(d)	(i)	resonance ✓	**2**
	(ii)	3300 (rev min^{-1}) ✓	
		Total	**10**